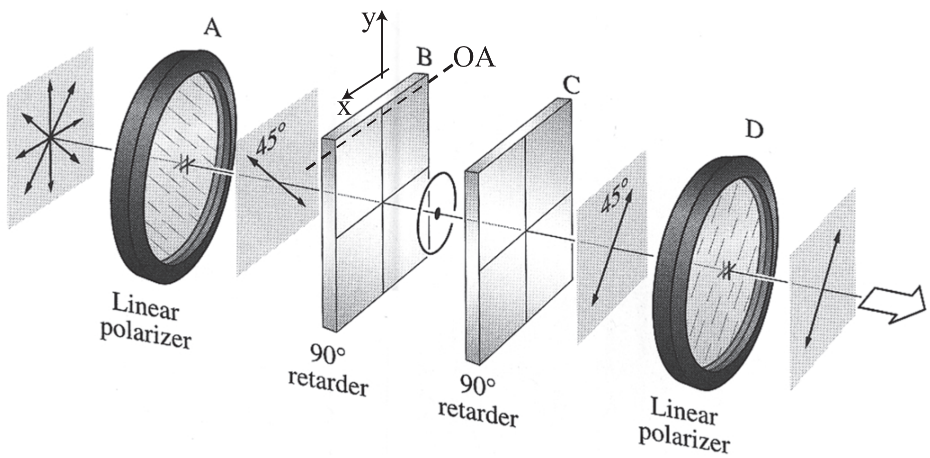
1. Matrix Representation of Polarization Changes (4 pts)

Describe in matrix representation the polarization state of light at various positions in the optical set-up sketched in the figure, where $n_\perp > n_\parallel$ in both retarders and the OA in *C* has the same orientation as indicated for *B*.

(a) Between *A* and *B*

(b) Between *B* and *C* – how is the polarization state affected by the 90° retarder *B*?

(c) Between *C* and *D* – how is it changed by the second retarder *C*?



(a) Between *A* and *B*:

$$\vec{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$ represents linear polarization at +45°.

(b) Between *B* and *C*: $n_\perp > n_\parallel$ implies $v_\perp \equiv v_y < v_x \equiv v_\parallel$. The *x* component of the beam travels faster than the *y* component. Therefore, E_x has a positive phase shift against E_y.

$$\vec{M} = \begin{bmatrix} e^{i\pi/4} & 0 \\ 0 & e^{-i\pi/4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\pi/2} \end{bmatrix} e^{i\pi/4} = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} e^{i\pi/4}$$

$$\vec{E} = e^{i\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -i \end{bmatrix} e^{i\pi/4}$$. This represents right-hand circular polarization.

(c) Between *C* and *D*:

$$\vec{E} = e^{i\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} \begin{bmatrix} 1 \\ -i \end{bmatrix} e^{i\pi/4} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{i\pi/2}$$. This represents linear polarization at –45°.

2. Polarizer and Retarder **(2 pts)**

You are given the polarizer and retarder, *A* and *B*, in problem 1, but you don't know which is which. How do you find out by using naturally polarized (*i.e.*, "unpolarized") light?

Both affect the transmitted irradiance differently. The polarizer reduces the irradiance of the unpolarized beam to half of its value (which now is, of course, polarized).

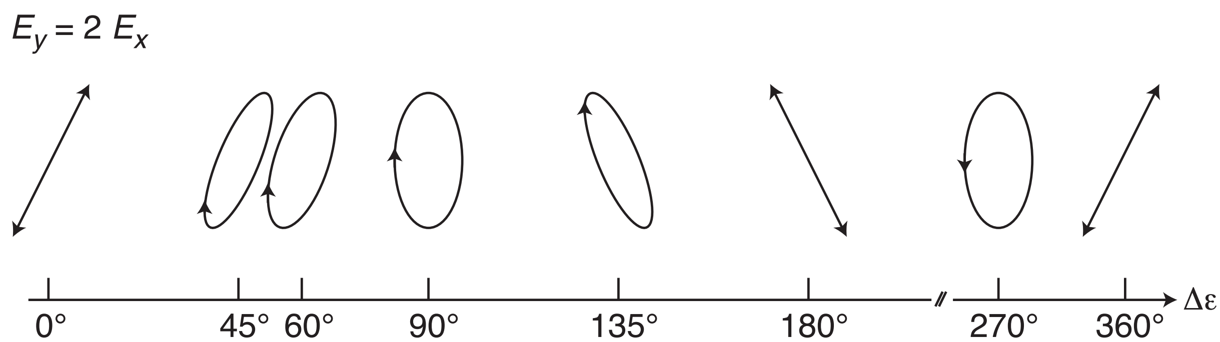
The wave plate doesn't block any *E* component; instead it shift the phase of one vs. the other. The irradiance remains unaffected.

3. Elliptically Polarized Light **(6 pts)**

A ray contains *E* components with both $E_x > 0$ and $E_y > 0$ and $E_x \neq E_y$. Describe verbally and sketch what you expect for the polarization state of that ray if the relative phase angle $\Delta\varepsilon = (\varepsilon_y - \varepsilon_x)$ is continuously shifted from 0° to 45°, 60°, 90°, 135°, 180°, 225°, 270° and 315°. How does the ratio E_x/E_y affect the picture?

At 0°, 180°, and 360°, obtain linear polarization. At 0° = 360°, the polarization angle depends on E_x/E_y as $\alpha = \arctan(E_y/E_x)$ with α measured from the *x* axis. At 180°, $\alpha = -\arctan(E_y/E_x)$.

At 90° and 270°, the polarization state is elliptical with the main axes of the ellipse pointing into the *x* and *y* directions. (For $E_x = E_y$, this is circular polarization.) At all other angle, the ellipse shifts orientation of its semimajor axis, with the eccentricity increasing as Δε gets closer to 0° or 180°. Between 0° and 180°, the handedness of the elliptic polarization is opposite to that between 180° and 360°.



4. Separation of Polarization States in Calcite (3 pts)

A calcite plate is cut such that the OA is parallel to the front face, and vertically oriented. A beam (λ_0 = 589 nm) impinges horizontally at an angle of incidence, $\varphi = 50°$. What is the angular separation of the *o*-ray ($n_o = 1.6584$) and the *e*-ray ($n_e = 1.4864$) within the plate?

Because the OA is parallel to the face of the plate, the *e* and *o* rays both experience the extremes of index separation, *i.e.*, they travel along the main axes of the wave front ellipsoids. Snell's law:

$$\sin\theta_i = n_{o/e}\sin\theta_{to/te}$$

$$\sin\theta_{to} = \frac{\sin\theta_i}{n_o} = \frac{0.766}{1.6584} = 0.462 \text{ ; } \theta_{to} = 27.5°$$

$$\sin\theta_{te} = \frac{\sin\theta_i}{n_e} = \frac{0.766}{1.4864} = 0.515 \text{ ; } \theta_{to} = 31.0°$$

$\Delta\theta = 3.5°$.

5. Application of the Fresnel Equations: Reflection from Cover Slide (5 pts)

Unpolarized light hits a microscope cover slide ($n_g = 1.5$) at an incident angle of 30° against the surface normal. What is the degree of polarization, $V = \dfrac{I_p}{I_p + I_n}$ where I_p and I_n are the irradiances of polarized and "natural" (*i.e.*, unpolarized) light, after the reflection? (Refer to Hecht, section 8.6.1, which you should have studied in your reading assignment for today.)

Snell's law: $\sin\theta_t = \dfrac{\sin\theta_i}{n_g}$; $\theta_t = 19.5°$

The reflected irradiance of the || component is matched by a part of the irradiance of the reflected ⊥ component. Together, these two irradiances form the unpolarized proportion I_n of the reflected beam. The excess || irradiance forms the polarized proportion I_p of the reflected beam.

$$R_\perp = \frac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)} = 0.057 \text{ ; } R_\parallel = \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)} = 0.025$$

$$V = \frac{I_p}{I_p + I_n} = \frac{R_\perp - R_\parallel}{R_\parallel + R_\perp} = 0.39$$, *i.e.*, the reflected beam is approx. 39% polarized.